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Exact boundary conditions in the age theory

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INTRODUCTION

In many problems of neutron slowing-down [1], which have been solved by means of the age theory, the condition has been used that the slowing-down density is zero at the boundaries.
As it is well known, the exact condition is that the total neutron current incident on the boundary is zero.
The distance from the boundary at which the slowing-down density is zero (the extrapolated end point) can be estimated to be [2]:

$$(2/3)\, L(u)/[1 - <\cos\Theta>]$$

where $L(u)$ is the total mean free path, $u = \log(E_0/E)$, E_0 is the initial energy of neutrons, E is the energy at any successive time, and $<\cos\Theta>$ is the average of the cosine of the angle of deflection produced by one collision (measured in the aboratory system).
Then, if the pile is very small and the slowing-down medium is so that $L(u)/[1 - <\cos\Theta>]$ is appreciable, the use of exact boundary conditions is of some importance.
Immediately, we find the slowing-down density for a hollow cylinder of infinite height with exact boundary conditions at both cylindrical surfaces.

SLOWING-DOWN PROBLEM

We consider the age theory without capture and time variation. The corrections to be made to obtain the solution of the complete problem are well known [1].

Let be a hollow cylinder given in cylindrical coordinates by:

$$a \leq r \leq b \quad ; \quad -\infty < z < +\infty \quad ; \quad 0 \leq \varphi \leq 2\pi$$

The angle independent slowing-down density $\chi(r, z, \theta)$ satisfies the age differential equation:

$$\partial\chi/\partial\theta = 1/r\partial(r\,\partial\chi/\partial r)/\partial r + \partial^2\chi/\partial z^2 + T(r, z, \theta) \quad (1)$$

The « simbolic age » θ is defined by:

$$\theta = 1/3 \int_0^u L^2(u')\, du'/\xi\,[1 - \langle\cos\Theta\rangle]$$

and the function $T(r, z, \theta)$ is related to the source function $S(r, z, u)$ by the equation:

$$T(r, z, \theta) = 4\pi S(r, z, u)\, \partial u/\partial\theta$$

The exact boundary conditions are:

$$[\chi(r, z, \theta) + k\,\partial\chi(r, z, \theta)/\partial r]_{r=a} = [\chi(r, z, \theta) - k\,\partial\chi(r, z, \theta)/\partial r]_{r=b} = 0 \quad (2)$$

where: $k = 2/3\, L(u)/[1 - <\cos\Theta>]$ is just the distance between the extrapolated end point and the boundary.
We shall use the « initial » condition: $\chi(r, z, 0) = 0$.
Let be the integral transform [3]:

$$\bar{\chi}(n, z, \theta) = \int_a^b r\,\chi(r, z, \theta)\, S_0(k, -k, \mu_n r)\, dr \quad (3)$$

whose inversion theorem is:

$$\chi(r, z, \theta) = \sum_{n=1}^{\infty} \bar{\chi}(n, z, \theta)/C_n \cdot S_0 \, (k, \, -k, \, \mu_n r)$$

with:

$$S_0(k, -k, \mu_n r) =$$
$$= J_0(\mu_n r)\Big[Y_0(k, \mu_n a) + Y_0(-k, \mu_n b)\Big] -$$
$$- Y_0(\mu_n r)\Big[J_0(k, \mu_n a) + J_0(-k, \mu_n b)\Big];$$

$$C_n = \Big[r^2/2 \Big\{ \tilde{S}_0(k, -k, \mu_n r) -$$
$$- \tilde{S}_{-1}(k, -k, \mu_n r)\, \tilde{S}_1(k, -k, \mu_n r) \Big\} \Big]_a^b$$

and:

$$\tilde{S}_q(k, -k, \mu_n r) =$$
$$= I_q(\mu_n r)\Big[Y_0(k, \mu_n a) + Y_0(-k, \mu_n b)\Big] -$$
$$- Y_q(\mu_n r)\Big[I_0(k, \mu_n a) + I_0(-k, \mu_n b)\Big]$$

The μ_n are the positive roots of the eigenvalue equation:

$$J_0(k, \mu a)\, Y_0(-k, \mu b) - J_0(-k, \mu b)\, Y_0(k, \mu a) = 0$$

Using the integral transform (3) and taking into account (2), the equation (1) is reduced to (see ref. 3):

$$\partial \bar{\chi}(n, z, \theta)/\partial\theta = - \mu_n^2\, \bar{\chi}(n, z, \theta) +$$
$$+ \partial^2 \bar{\chi}(n, z, \theta)/\partial z^2 + \bar{T}(n, z, \theta)$$

This equation can be solved by standard methods. Finally, we obtain the slowing-down density:

$$\chi(r, z, \theta) =$$
$$= 1/2\sqrt{\pi} \sum_{n=1}^{\infty} 1/C_n \int_0^{\theta} \exp\Big[-\mu_n^2(\theta - s)\Big] \cdot$$
$$\cdot \int_{-\infty}^{\infty} (\theta - s)^{-1/2} \exp\Big[-(z-\eta)^2/4(\theta - s)\Big] \cdot$$
$$\cdot \bar{T}(n, \eta, s)\, ds\, d\eta \cdot S_0(k, -k, \mu_n r)$$

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