COOPERATIVE GAME THEORY SOLUTION IN AN UPSTREAM-DOWNSTREAM RELATIONSHIP

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Abstract

What happens when a firm does not want to be vertically integrated? Within a buyer-supplier relationship, and in a context of specific investments, we analyze firm interactions when they decide to cooperate instead of becoming vertically integrated. We develop a model that presents an alternative to vertical integration with firms that cooperate with each other under incomplete contracts. The contribution of our approach is twofold: first, a mathematical component, where we show how to apply the maximin to compute the characteristic functions of the model; and second, an economic element, where we reconsider the upstream-downstream relationship under a cooperative framework and no integration. We find that the total generated value is Pareto optimum if firms cooperate and distribute the benefits following the Shapley value solution. Finally, the Shapley value measures the power that each firm has in the bargaining process.

KEYWORDS: cooperative game, Shapley value, core, incomplete contracts, specific investments, upstream-downstream relations, characteristic functions.

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INTRODUCTION

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The theory of the firm deals with the boundaries of a firm and it also focuses on the coordination and motivation problems faced by the different participants. The first definitions of a firm were economic models that considered the firm as a black box where physical inputs and labor came out in an output, at minimum cost and maximum profit (Hart, 1995). This definition had important limitations that gave place to new theories of the firm. Coase (1937) is recognized as the first researcher to provide an explanation about the existence of firms. He suggested that there were market imperfections that generated transaction costs and, in this context, the presence of firms may help to alleviate these market failures. In particular, this author argued that, under some circumstances, transactions could be done through an organization at a lower cost rather than through markets. Later, in 1972, Alchian and Demsetz defined the firm "as a nexus of contracts" and they also provided a description of the "classical capitalist firm".

In 1975 Williamson took the concept of transaction costs given by Coase, and he elaborated it further. In fact, Williamson redefined the concept of transaction costs and applied it to different forms of organizations, not just the market, making a substantial progress in the study of the firm. His most important contribution was to elucidate the roles of imperfect contracts and specific investments (asset specificity, human specificity). Another important contribution was the fact of comparing the transactional efficiency of alternative governance structures, including vertical integration, non-standard contracts and relational contracts (Gibbons, 2000)

Williamson's works opened the door to new investigations based on the ownership of the physical assets. Within this group, one can find the seminal work of Grossman and Hart (1986), who defined the firm as "a collection of physical assets that are jointly owned" (Zingales, 1998).

Although the ownership of physical assets has been used to explain the boundaries of the firm and vertical integration serves as a mechanism to save transaction costs, another important problem remains: the specificity of human assets. The problem of specific human assets in a context of vertical integration was already treated by Klein et al in 1978, at the time of describing the problems between General Motors (GM) and Fisher Body (FB). Human assets cannot be acquired because they are inherent to human beings. Klein revisited the vertical integration between General Motors and Fisher Body and argued that the costs associated with vertical integration are generally incentive costs that are unrelated to the degree of specific investments. Therefore, vertical integration becomes the most plausible solution when the degree of specific investments becomes high (Klein, 1988). Vertical integration avoids transaction costs when the costs are associated with physical assets, but it does not explain what happens when vertical integration is linked to specific human assets.

Rajan and Zingales (2000) indicate that General Motors was, and still is, a vertically integrated firm, which controls the physical assets through ownership. However, they argue the nature of the firms is changing. Large conglomerates have been broken up and their units have been spun-off as stand-alone companies. One example is Nucor, a steel manufacturer that abandoned the tradition of backward integration and out-sourced the entire supply chain of raw material (Holmström and Roberts, 1998).

As a result, vertical integration does not seem to be a good solution any more under certain circumstances. So we wonder if there is another way, different from vertical integration, to solve the problems. In particular, we analyze what happens when some firms decide to cooperate instead of being vertically integrated in a scenario with the presence of specific investments. More specifically, what happens if a supplier does not accept to be vertically integrated and it prefers, instead, to deal with several customers? To answer these questions, we develop a model that attempts to bring a new alternative solution to the problem, a model where the firms negotiate and cooperate to each other in line with the observed practices of many companies.

The contribution of this model is twofold: first, a mathematical contribution where we show how to apply the maximin that is included in the Minimax Theorem (see Marchi, 1967) to compute the characteristic functions of the model; and second, an economic contribution to solve a supplier-manufacturer relationship. Doing this, we present an alternative way to look at the upstream-downstream relationships under a cooperative framework and incomplete contracts.

We find that, under cooperation, the total value generated is Pareto optimum when they cooperate and distribute the benefits following the Shapley value solution. In fact, the Shapley value measures the power that the different participants have in the bargaining process. The bargaining power under cooperative game theory is determined by which player is needed the most (Brandenburger, 2007). In our model, the bargaining power is given by the specificity of the investment. The result of our model is that the supplier's bargaining power is what makes the cooperation between firms possible, making such upstream-downstream relationship enforceable. And, as a consequence, the first best that maximizes the total welfare can be achieved.

This paper proceeds as follows: in section 1, we present the literature review, while in section 2, the economic environment of the model is described. We develop a model under an incomplete and cooperative framework, being the core and the Shapley value the solutions to such model. Finally, in section 3, we present the conclusions of the model, followed by a description of some limitations and future research avenues.

1- LITERATURE REVIEW

The firm, as a black box where physical inputs and labor came out in an output, at minimum cost or maximum profit, has been the basic model of a firm for two hundred years. However, this theory presented important limitations because it takes into account neither the incentives inside the firm, nor its internal structure. In addition, it did not establish the boundaries of the firm. These limitations gave place to the appearance of new theories of the firm. Coase (1937) was the first researcher to give an explanation about the existence of the firms. He suggested that the markets were imperfect and firms existed to solve these market failures. Coase also dealt with the internal organization and suggested that firms would exist only in an environment in which they could perform better than markets (Gibbons, 2000). Furthermore, he argued that, under some circumstances, transactions can be made through the organization at a lower cost rather than through markets. This is so because the organizations ameliorate some market failures and they have the advantage of using employment contracts as a way to save transaction costs.

Later, in 1972, Alchian and Demsetz defined a firm “as a nexus of contracts”. They said a firm is characterized by something more than legal status, namely the technology of team production, which means production with an inseparable production function. Thus, they introduced the free-riding problem in team productions. Their solution was to propose the presence of a monitor, who had the right to fire and hire the members of the team and, by doing that, they described the “classical capitalist firm”

In 1975 Williamson took the concept of transaction costs given by Coase and redefined it. In fact, Williamson introduced the transaction cost approach to the study of organizations, suggesting a new way of thinking about the firm and focusing on the analysis of transactions and contracts. In particular, he said that the transaction costs approach relies on two assumptions concerning the individuals: 1) human agents are subject to bounded rationality, and 2) at least some agents may behave in opportunistic way (Williamson, 1981). Furthermore, the characteristics of the transactions, such as their frequency, the presence of specific assets and the presence of information asymmetries may be important sources of transaction costs. Economic exchanges could be efficiently organized by contracts but, due to bounded rationality, the nature of the transactions and the opportunistic behavior of the individuals, contracts are often incomplete.

Williamson made a substantial progress in understanding the nature of the firm, and one key contribution was to elucidate the roles of imperfect contracts and specific investments (asset specificity and human specificity). Asset specificity is critical because, once the investment is made for a determined transaction between a buyer and a supplier, the value of this asset becomes lower if it used in a different transaction. Then, the supplier is “locked into” the transaction (Williamson, 1981). This also applies to a buyer when the buyer cannot make transactions with others suppliers.

Another important contribution was to compare the transactional efficiency of alternative governance structures, including vertical integration, non-standard contracts and relational contracts (Gibbons, 2000). An example of vertical integration was treated by Klein et al in 1978, explaining the relationship between General Motors (GM) and Fisher Body (FB). General Motors signed a contract with Fisher Body in which GM argued to purchase all its closed bodies from FB for 10 years in 1919, and they agreed on a price for delivering based on a cost plus a margin, that also included provisions that GM would not be charged more than other rival automobile manufacturers. As it is well-known, GM faced a much higher demand than the forecasted one, increasing its dependence on FB. As a response to this situation, GM was unsatisfied with the price settled and proposed FB to locate its body plants near the GM assembly plants, an offer that FB refused. As a consequence, GM started to acquire FB stock in 1924 and, finally, it completed a merger agreement in 1926. A GM car needed specific investments, and site specificity implied transaction costs. This vertical integration between GM and FB solved the problems associated with the presence of uncertainty in demand and costs.

Chandler (1977) and Porter and Livesay (1971) analyzed forward integration into distribution activities for the case of manufacturing firms. In retailing, vertical integration was made for those commodities that needed a considerable number of sale information points. In this context, specific human assets were needed to provide service and achieve larger sales, and the integration into wholesaling occurred for those commodities that were perishable and branded. The explanation for the existence of forward integration is that the manufacturer’s reputation was at risk: contracts to turn on inventories or to destroy stocks were neither self-enforcing nor incentive compatible. As Williamson (1981) explained “the commodities that had none of these characteristics (perishable and branded) were sold through market distribution channels where no special hazards were posed”.

Williamson’s analysis encouraged further research based on the ownership of physical assets. The seminal work of Grossman and Hart (1986), who defined the firm as “a collection of physical assets that are jointly owned”, is based on

Williamson's approach, and along with Moore (1990) these authors developed a theory of incomplete contracts and property rights. There, it is this notion of property rights what determines the right to take decisions concerning all the issues that are not explicitly covered in the contract (Zingales, 1998).

After this, when explaining the boundaries of the firm, most researchers have assumed that the ownership of physical assets confers the right to take decisions. This means that the ownership of physical assets has been used as an incentive mechanism or a control mechanism to induce the agent to make the optimal effort or to carry out the optimal investment.

Our point is that even when ownership of physical assets is used to explain the boundaries of the firm and vertical integration can save transaction costs, another problem still remains: the specificity of human assets. As Klein (1988) pointed out, human assets cannot be acquired because they are inherent to a human being. In fact, this author revisited the General Motors and Fisher Body case and argued that the costs associated with vertical integration are generally incentive costs unrelated with the degree of the specific investments. Therefore, the vertical integration is the most possible solution while the degree of specific investment is high (Klein, 1988). So, vertical integration avoids the transaction costs when the costs are associated with physical assets but, what happens when vertical integration occurs in a context with specific human assets? In this case, vertical integration will not solve the transaction costs problem because the specific human capital is inherent to those human beings and it cannot be owned by a third party. Thus, the opportunistic behavior present in the original organizational arrangement is not eliminated and the benefits of a vertical integration process remain unclear.

In fact, if the conflict between General Motors and Fisher Body was only based on a hold-up problem of the investments in physical assets and specific human assets did not matter, the best solution would have been the acquisition by GM of the Fisher Body's physical assets. But vertical integration, meaning that the Fisher brothers become employees instead of independent contractors, does not eliminate the potential hold-up. Ownership of the specific human asset was still belonging to the Fisher brothers. So, as Klein pointed out, the solution was that General Motors did not buy the closed bodies from Fisher Body, but General Motors had to produce the closed bodies with the assistance of Fisher Body. General Motors was the owner of the plants and could determine where the plants were located, but the Fisher brothers became managers with the capacity to control or hold up GM with regard to specific investments in human assets.

This vertical integration implied that General Motors transformed not only the Fisher brothers into employees but also all the employees of Fisher Body into GM employees. General Motor stopped buying to start producing, acquiring in this way the organizational capital of Fisher Body Corporation. GM became the owner of all employment contracts and also the owner of the knowledge concerning the production of closed bodies. It is in this way that General Motors owned the specific human asset (Klein, 1988).

Furthermore, this solution was a successful one because the number of employees was huge. If few employees are involved, they can leave the firm and the organizational capital will be gone with them. But the threat that all the employees will leave the firm is not credible when the number of employees is so huge. After the vertical integration process, the Fisher brothers could not threat General Motors saying that all their former employees would leave the firm (taking their knowledge with them). In this sense, vertical integration in organizations with large human teams implies the ownership of the human asset as it happens with physical assets. It is in this sense that Klein mentions that specific human capital can be acquired and that the vertical integration could be a solution under some circumstances.

Rajan and Zingales (2000) indicate that General Motors was, and still is, a vertically integrated firm, that controls the physical assets through ownership. However, as these authors point out, the nature of many firms is changing: "Large conglomerates have been broken up and their units have been spun-off as stand-alone companies. Vertically integrated manufacturers have relinquished direct control of their suppliers and have moved toward looser forms of collaboration" (Rajan and Zingales, 2000). One example of this is Nucor, a steel manufacturer that abandoned the tradition of backward integration and out-sourced the entire supply chain of raw material (Holmström and Roberts, 1998). Even General Motors is changing and it has facilitated the spin off of its major part supplier, Delphi. Toyota, and the relationship with its suppliers, provides another example of new forms of organization to deal with specific assets. In fact, Aoki (1990) talks of quasi-integration to refer to these links between Toyota and its suppliers which remain independent legal entities but exchange information and closely cooperate in the production process and the development of new products.

In summary, vertical integration seems not to be a good solution under some circumstances, as many researchers have already pointed out. This fact makes us wonder if there is another way to solve these problems that vertical integration does not address. In particular, we ask ourselves: *what happens if the firms decide to cooperate instead of being vertically integrated under the existence of specific investments? Additionally, what happens if the supply firm refuses to be vertically integrated?* To answer these questions, we develop a model attempting to bring a new alternative solution to vertical integration. We propose a model where the firms cooperate with each other and we find that the total value generated, under cooperation, is Pareto optimum if they distribute the benefits following the Shapley value solution.

We proceed now to mention briefly some important features of the Cooperative Game theory, before describing the model.

1.1- COOPERATIVE GAME THEORY

The game theory of an arbitrary set of players or agents is created by John von Neumann and Oskar Morgenstern in 1944[1]. The game theory is the study of games, also called strategic situations (Serrano, 2007). The game theory is divided in two main approaches: the cooperative and the non-cooperative game theory. The actors in non-cooperative game theory are individual players who

[1] Morgenstern, O., Neumann, von, J., 1944. Theory of Games and Economic Behavior. Princeton University Press.

may reach agreements only if they are self-enforcing, while in cooperative game theory, the actors are coalitions, group of players. As Serrano (2007) points out, the fact that a coalition has formed and that it has a feasible set of payoffs available to its members is now taken as given.

These two approaches of game theory imply two different forms to look at the same problem. As Aumann (1959) puts it, “the game is one ideal and the cooperative and non cooperative approaches are two shadows”. Game theory models imply situations in which players make decisions to maximize their own utility, while the rest of the players do the same. The decisions of the latter affect each other utilities. Cooperative game theory looks for the possible set of outcomes, study what the players can achieve, which coalitions will be formed, how the coalitions will distribute the outcomes and whether the outcomes are robust and stable (Sosic and Nagarajan, 2006).

The terms of non-cooperative game theory may, mistakenly, suggest that there is no place for cooperation, and the term of cooperative game theory might suggest that there is no room for conflict or competition. But as Brandenburger (2007) has already pointed out, neither is the case. Part of the non-cooperative game theory studies the possibility of cooperation in ongoing relationships, while many papers that use cooperative game theory also include the possibility of competition among players.

We follow a cooperative game approach that consists of two elements: 1) a set of players and 2) a characteristic function specifying the value created by different subsets of the players involved in the game. The cooperative game theory attempts to answer how the total value is divided up among the players and this answer will depend on their bargaining power. Furthermore, in cooperative game theory, a player’s bargaining power depends on how much other players need him to form coalitions, or his marginal contribution. Brandenburger defines it as follows: ”the marginal contribution of a particular player is the amount by which the created overall value would shrink if this particular player leaves the game”. It is in this way that cooperative game theory captures the idea of competition among different players in bargaining situations.

There are different ways of solving cooperative games and we want to emphasize two of them: the core and the Shapley value. The core was first proposed by Francis Ysidro Edgeworth in 1881, and it was later reinvented and defined in game theoretic terms by Gillies (1953). The core is a solution concept that allocates to each cooperative game the set of payoffs that no coalition can improve upon or block (Serrano, 2007). Concerning the Shapley value, it was first proposed by Lloyd Shapley in his 1953 PhD dissertation. This solution prescribes a single payoff for each player, which is the average of all marginal contributions of that player to each coalition he or she is a member of (Serrano, 2007).

2- THE ECONOMIC ENVIRONMENT

This model is developed under a cooperative game framework and the presence of incomplete contracts. Players take their decisions of buying and producing, depending on their expected benefits. To compute those expected benefits, we have chosen the Shapley value solution because it takes into account what each player could reasonably get before the game starts.

More specifically, we consider a model with three agents, two downstream parties and one upstream party. The upstream party is a supply firm which sells its product to the downstream parties. The supplier has to make specific investments to obtain the product q, which has no alternative use out of these relationships. We assume the existence of a parameter, λ, that measures the degree of investment specificity, where $\lambda \in [0,1]$. The higher the value of λ, the more specific the investment. If $\lambda = 1$, the investment is fully specific, while a value $\lambda = 0$ indicates that the investment becomes totally general. Furthermore, this product q requires a quality process that is not observable to third parties and, therefore, it is not possible to write down an enforceable contract. The supplier gets this q at a cost λc_s.

The downstream parties buy the product from the supplier and they sell it in different markets. Each firm is a monopoly in its market. These downstream firms have incentives to merge or to be vertically integrated with the supplier because this would ensure them the required quality. So we can say the downstream firms compete for the supplier giving him bargaining power, in the sense that both downstream firms need the supplier. In this model, the bargaining power responds to the existence of specific investments. As a result, the incentive problem is twofold: on one hand, the downstream firms want the supplier to produce under a given quality process. On the other hand, the supplier, after having made the specific investments, faces the possibility that the downstream firms may argue that the product has not reached the required quality and, consequently, they will pay less for the product or, in the worst case, they will refuse to buy it.

We consider a one-shot supply transaction. The downstream firm 1 buys q_1 to the supplier and the second firm buys q_2, such that $q_1+q_2=q$ and $q_1, q_2 \quad 0$. The supplier has to make specific investments to produce these quantities at a cost $\lambda c_{s1}, \lambda c_{s2}$ and sells them at prices w_1, w_2, being $w_1, w_2 \geq 0$ and $w_1 \in \left[0, \bar{w}_1\right]$ and $w_2 \in \left[0, \bar{w}_2\right]$

The downstream parties buy the quantities at prices w_1, w_2 and sell them at p_1, p_2, respectively. The prices p_1, p_2 are positive and $p_1 \in \left[0, \bar{p}_1\right]$, $p_2 \in \left[0, \bar{p}_2\right]$ and they are decided by each firm. The downstream firm 1 faces a demand[2] function given by $q_1 = f_1(p_1) = \beta_1 - \gamma_1 p_1$ and the downstream firm 2 faces the demand function $q_2 = f_2(p_2) = \beta_2 - \gamma_2 p_2$. For simplicity, we assume linear demand functions. Additionally, to guarantee that the inverse demand functions exist and they are well defined, we further assume that:

1. the demand functions have negative slopes

[2] For simplicity, it is assumed the demand function is known by the downstream parties. We know this is a strong assumption but if we assume expected demand functions, our results will remain unchanged.

2. f_1, f_2 are differentiable functions whose first derivatives are strictly negative and finite for any $p_1 \in \left[0, \bar{p}_1\right]$ and $p_2 \in \left[0, \bar{p}_2\right]$ such that $f_1, f_2 \quad 0$.

We also assume that $0 \leq w_1 \leq p_1 \leq \frac{\beta_1}{\gamma_1}$ and $0 \leq w_2 \leq p_2 \leq \frac{\beta_2}{\gamma_2}$

The players of the model have transferable utilities and they are rational, so they take decisions that maximize their expected utilities. The respective utility functions are: $U_s = U\left(\Pi_s\right) + \varepsilon_s$ for the supplier, $U_1 = U\left(\Pi_1\right) + \varepsilon_1$ for the downstream firm 1 and $U_2 = U\left(\Pi_2\right) + \varepsilon_2$ for the downstream firm 2. Furthermore, $\varepsilon_s, \varepsilon_{1,} \varepsilon_2$ are random variables that follow a normal distribution with $E\left(\varepsilon_s\right) = E\left(\varepsilon_1\right) = E\left(\varepsilon_2\right) = 0$ and variances $\sigma_{s}2, \sigma_{1}2, \sigma_{2}2$, respectively. For simplicity, we also assume the different players are risk neutral.

Finally, the players' expected benefits are given by the following equations.

$\Pi_1 = R_1\left(q_1, p_1\right) - w_1 q_1$, for the downstream firm 1, where the revenue is equal to $R_1\left(q_1, p_1\right) = p_1 * q_1$

$\Pi_2 = R_2\left(q_2, p_2\right) - w_1 q_2$, for the downstream firm 2, where $R_2\left(q_2, p_2\right) = p_2 * q_2$, and

$\Pi_3 = q_1 w_1 + q_2 w_2 - \lambda c_{s1} q_1 - \lambda c_{s2} q_2$, for the supplier.

2.1 -THE TIMING

As we mentioned earlier, the players decide to buy and produce in terms of their expected benefits. The timing of the model becomes as follows:

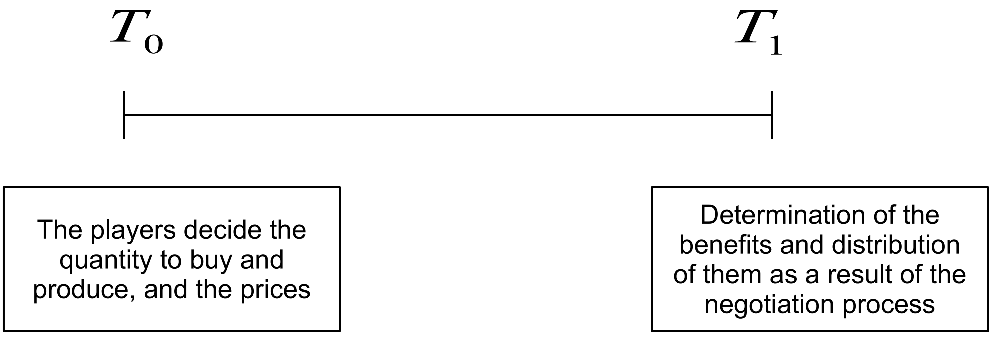

Figure 1: Timing of the Model

At time 0, the downstream firms decide the quantity of product they will buy and the prices they are going to sell it. The supplier decides to produce this level of product, along with the quality and the price it will charge to the downstream parties. At time 1, the three firms will determine the payoffs and the distribution of the benefits according to the negotiation process. Due to the existence of incomplete information and the framework of incomplete contracts, the players decide output level and their prices as a function of their expected future benefits and their distribution

2. 2- BENCHMARK

Under perfect information and complete contracts, we compute the first best. To compute the first best it is assumed there is one player who has perfect information and is the owner of the assets. The firms act as one player who maximizes the total expected benefits (Π_T). The equation of the total expected benefit is:

$\Pi_T = p_1 q_1 - \lambda c_{s1} q_1 + p_2 q_2 - \lambda c_{s2} q_2$, being the demand functions

$q_1 = f_1(p_1) = \beta_1 - \gamma_1 p_1$ and $q_2 = f_2(p_2) = \beta_2 - \gamma_2 p_2$.

To assure the presence of a maximum, it must be satisfied that

$$f_x = f_y = 0$$
$$f^{2}_{xx} f^{2}_{yy} - 2f^{2}_{xy} > 0$$
$$f^{2}_{xx} < 0$$

So, the maximum is achieved.

$$\frac{\partial \Pi_T}{\partial p_1} = \beta_1 - 2\gamma_1 p_1 + \lambda c_{s1} \gamma_1 = 0$$

$$\frac{\partial \Pi_T}{\partial p_2} = \beta_2 - 2\gamma_2 p_2 + \lambda c_{s2} \gamma_2 = 0$$

From these derivatives, we get the prices and quantities that maximize the total net profit.

$$p_2 = \frac{\beta_2 + \lambda c_{s2} \gamma_2}{2\gamma_2} \quad \text{and} \quad q_2 = \frac{\beta_2 - \lambda c_{s2} \gamma_2}{2}$$

$$p_1 = \frac{\beta_1 + \lambda c_{s1} \gamma_1}{2\gamma_1} \quad \text{and} \quad q_1 = \frac{\beta_1 - \lambda c_{s1} \gamma_1}{2}$$

The total benefit in the benchmark case becomes:

$$\Pi_T = \frac{\left(\beta_1 - \lambda c_{s1} \gamma_1\right)^2}{4\gamma_1} + \frac{\left(\beta_2 - \lambda c_{s2} \gamma_2\right)^2}{4\gamma_2}$$

Next, we will compare this benchmark solution with the results obtained under incomplete contracts and a cooperative game framework.

2. 3- THE NON-COOPERATIVE OR NASH SOLUTION

Before showing the model under a cooperative game theory framework, we compute the solution under a non-cooperative game framework. To do this, we use the Nash equilibrium definition. In our model, we need:

$$\Pi_1\left(\bar{p}_1, \bar{p}_2, \bar{w}_1, \bar{w}_2\right) \geq \Pi_1\left(p_1, \bar{p}_2, \bar{w}_1, \bar{w}_2\right) \text{ for all } p_1 \quad \text{(A)}$$

$$\Pi_2\left(\bar{p}_1, \bar{p}_2, \bar{w}_1, \bar{w}_2\right) \geq \Pi_2\left(\bar{p}_1, p_2, \bar{w}_1, \bar{w}_2\right) \text{ for all } p_2 \quad \text{(B)}$$

$$\Pi_3\left(\bar{p}_1, \bar{p}_2, \bar{w}_1, \bar{w}_2\right) \geq \Pi_3\left(\bar{p}_1, \bar{p}_2, w_1, w_2\right) \text{ for all } w_1 \text{ and } w_2 \quad \text{(C)}$$

The equations of the model are the following:

$$\Pi_1\left(p_1, p_2, w_1, w_2\right) = \left(p_1 - w_1\right)\left(\beta_1 - \gamma_1 p_1\right) \quad (1)$$

for the downstream firm 1, where $p_1 \in \left[0, \frac{\beta_1}{\gamma_1}\right]$ and $w_1 \in \left[0, \frac{\beta_1}{\gamma_1}\right]$

$$\Pi_2(p_1, p_2, w_1, w_2) = (p_2 - w_2)(\beta_2 - \gamma_2 p_2) \quad (2)$$

for the downstream firm 2, where $p_2 \in \left[0, \frac{\beta_2}{\gamma_2}\right]$ and $w_2 \in \left[0, \frac{\beta_2}{\gamma_2}\right]$

$$\Pi_3(p_1, p_2, w_1, w_2) = (w_1 - \lambda c_{s1})(\beta_1 - \gamma_1 p_1) + (w_2 - \lambda c_{s2})(\beta_2 - \gamma_2 p_2) \quad (3)$$

for the supplier and $\lambda c_{s1} \leq w_1$ and $\lambda c_{s2} \leq w_2$.

To compute the Nash solution we take equation (1) and apply (A) looking for the value p_1 that maximizes equation (1), ceteris paribus.

The Nash solution to the downstream firm 1 is $p_1 = \frac{\beta_1 + \gamma_1 w_1}{2\gamma_1}$

We do the same for the downstream firm 2, and the value p_2 that maximizes the function Π_2 is $p_2 = \frac{\beta_2 + \gamma_2 w_2}{2\gamma_2}$

Similarly, for the supplier, we must calculate the values w_1 and w_2 that maximize equation (3) given the rest of variables (p_1, p_2) . The results are:

$$w_1 = \frac{\beta_1 + \lambda c_{s1} \gamma_1}{2\gamma_1} \qquad w_2 = \frac{\beta_2 + \lambda c_{s2} \gamma_2}{2\gamma_2}$$

Thus, once the prices w_1 and w_2 have been obtained, we replace them in p_1 and p_2 , and we get:

$$p_1 = \frac{3\beta_1 + \lambda c_{s_1} \gamma_1}{4\gamma_1} \qquad p_2 = \frac{3\beta_2 + \lambda c_{s_2} \gamma_2}{4\gamma_2}$$

Replacing this solution in the equations (1), (2) and (3), the players' net benefits become:

$$\Pi_1 = \frac{(\beta_1 - \lambda c_{s1} \gamma_1)^2}{16\gamma_1}$$

$$\Pi_2 = \frac{(\beta_2 - \lambda c_{s2} \gamma_2)^2}{16\gamma_2}$$

$$\Pi_s = \frac{\left(\beta_1 - \lambda c_{s1}\gamma_1\right)^2}{8\gamma_1} + \frac{\left(\beta_2 - \lambda c_{s2}\gamma_2\right)^2}{8\gamma_2}$$

As it can be seen, the Nash Solution shows us the effects of the double-marginalization. This is not an efficient result and, obviously, we do not reach the first best.

2. 4 - INCOMPLETE CONTRACTS AND COOPERATIVE GAME MODEL

Following the timing of the model, at time 0, the three players decide the quantity to sell and produce but these decisions will take into account the future benefits they expect to achieve at time 1. So, the players have to estimate their future benefits. To do this, we are going to use a cooperative game approach based on the subject of the Theorem of the Minimax created by von Neumann (1928). Under cooperative games, the players negotiate and compute their payoffs under different coalitions. The fact of using the maximin to compute the payoffs implies that, in the negotiation process, the actors estimate the least amount they can get if the other players play against them; in other words, they compute the payoffs in the worst scenarios. In order to compute the payoffs or the characteristic functions of the game, we must define first what a characteristic function is. The definition of a **characteristic function**[3] of an n-person game assigns to each coalition, which is a subset S of the players, the best payoff that each one can achieve without the help of other players. In other words, that is the value $v(S)$ that coalition S can guarantee for itself by coordinating the strategies of its members, no matter what the other players do. It is standard to define the characteristic value of the empty coalition, Φ , as 0 so $v(\Phi)=0$. The characteristic function implies that if X_s is the set of strategies available to the player in S , and Y_{N-S} is the set of strategies available to the players in $N-S$, then

$$v(S)=\max_{x\in X_s}\min_{y\in Y_{N-S}}\sum_{i\in S} e_i(x,y) ,$$

where $e_i(x,y)$ is the payoff to player i when x and y are the strategies played by the players of the parties S and $N-S$. This is for the mixed

[3] Thomas, L.C., 1984. Games, Theory and Applications. Ellis Horwood Limited

extension of finite games. It follows from this definition that if S and T are disjoint coalitions for finite n-person games, we get:

$v(S \cup T) \geq v(S) + v(T),\ if\ S \cap T = \{\Phi\}$.

That is, superadditivity.

Once the characteristic function is defined, we present the equations of the model:

$$\Pi_1(p_1, p_2, w_1, w_2) = (p_1 - w_1)(\beta_1 - \gamma_1 p_1) \qquad (1)$$

for downstream firm 1, where $p_1 \in \left[0, \frac{\beta_1}{\gamma_1}\right]$ and $w_1 \in \left[0, \frac{\beta_1}{\gamma_1}\right]$

$$\Pi_2(p_1, p_2, w_1, w_2) = (p_2 - w_2)(\beta_2 - \gamma_2 p_2) \qquad (2)$$

for downstream firm 2, where $p_2 \in \left[0, \frac{\beta_2}{\gamma_2}\right]$ and $w_2 \in \left[0, \frac{\beta_2}{\gamma_2}\right]$ and

$$\Pi_3(p_1, p_2, w_1, w_2) = (w_1 - \lambda c_{s1})(\beta_1 - \gamma_1 p_1) + (w_2 - \lambda c_{s2})(\beta_2 - \gamma_2 p_2) \qquad (3)$$

for the supplier and $\lambda c_{s1} \leq w_1$ and $\lambda c_{s2} \leq w_2$. The degree of investment specificity, λ , is high and it could be close to one in this model.

The possible coalitions of the model are:

$v(\varphi), v(1), v(2), v(3), v(1,2), v(1,3), v(2,3), v(1,2,3)$.

The computation for the characteristic function for the downstream firm 1 becomes as follows:

First, we take equation (1) and apply the maximin

$$v(1) = \max_{p_1} \min_{p_2, w_1, w_2} \Pi_1(p_1, p_2, w_1, w_2) = \max_{p_1} \min_{p_2, w_1, w_2} \left[p_1(\beta_1 - \gamma_1 p_1) - w_1(\beta_1 - \gamma_1 p_1)\right]$$

(4)

We must find the minimum values of p_2, w_1, w_2 that minimize this function for a given p_1 , and then we will maximize that function with respect to p_1 . In our context, the minimum value of w_1 that minimizes this function given p_1 is

$w_1 = \frac{\beta_1}{\gamma_1}$, so we replace this value in the function and look for the p_1 that maximizes the function.

$$v(1) = \max_{p_1} \left[p_1(\beta_1 - \gamma_1 p_1) - \frac{\beta_1}{\gamma_1}(\beta_1 - \gamma_1 p_1) \right] = \max_{p_1} \Pi_1(p_1, p_2, w_1, w_2)$$

$$\frac{\partial \Pi_1}{\partial p_1} = \beta_1 - 2\gamma_1 p_1 + \beta_1 = 0$$

To ensure the existence of a maximum, we check the second derivative

$\frac{\partial^2 \Pi_1}{\partial p_1{}^2} = -2\gamma_1$. It is negative and, therefore, we have got a maximum.

Thus, $p_1 = \frac{\beta_1}{\gamma_1}$ and $q_1 = 0$, and replacing these values in the function (4) we get that the characteristic function for the downstream firm 1 becomes:

$$v(\{1\}) = v(1) = 0 \ . \qquad (5)$$

This is the payoff that the player 1 will have in his worst scenario. It means that if the supplier charges him the highest price $w_1 = \frac{\beta_1}{\gamma_1}$, which is the highest cost that the downstream firm 1 can face, the downstream firm 1´s best response is to set the highest price of $p_1 = \frac{\beta_1}{\gamma_1}$ where the demand quantity is equal to 0. In that case, he gets zero profits.

The steps to compute the characteristic function to the downstream firm 2 are similar to the ones already computed for downstream firm 1. We proceed to simply write down the results and present the equation (6).

$$v(2) = \max_{p_2} \min_{p_1, w, w_2} \Pi_2(p_1, p_2, w_1, w_2) = \max_{p_2} \min_{p_1, w_1, w_2} \left[p_2(\beta_2 - \gamma_2 p_2) - w_2(\beta_2 - \gamma_2 p_2) \right]$$

$p_2 = \frac{\beta_2}{\gamma_2}$ and $q_2 = 0$

The characteristic function or the payoff that downstream firm 2 will receive in case that the other players join themselves against him becomes

$$v_2(\{2\}) = v(2) = 0 \qquad (7)$$

Equations (5) and (7) or the characteristic functions for the downstream firm 1 and the downstream firm 2, indicate the lowest value that they are able to get under the worst scenario.

Similarly, for the supplier, the characteristic function becomes:

$$v(3) = \max_{w_1, w_2} \min_{p_1, p_2} \Pi_3(p_1, p_2, w_1, w_2) =$$

$$\max_{w_1, w_2} \min_{p_1, p_2} \left[w_1(\beta_1 - \gamma_1 p_1) + w_2(\beta_2 - \gamma_2 p_2) - \lambda c_{s1}(\beta_1 - \gamma_1 p_1) - \lambda c_{s2}(\beta_2 - \gamma_2 p_2) \right]$$

We have to look for the minimum values of p_1, p_2 that minimize the function. These values are:

$$v(3) = \max_{w_1, w_2} \left[(w_1 - \lambda c_{s1}) \left(\beta_1 - \gamma_1 \frac{\beta_1}{\gamma_1} \right) + (w_2 - \lambda c_{s2}) \left(\beta_2 - \gamma_2 \frac{\beta_2}{\gamma_2} \right) \right]$$

$$v(3) = \max_{w_1, w_2} [0]$$

The characteristic function for the supplier is:

$$v(\{3\}) = v(3) = 0 \qquad (8)$$

The equation (8) is telling us the value that, in the worst case, the supplier gets. We can think that this is the case that the downstream firms join together and argue that the quality is not the one required and they do not want the product. The supplier´s payoff is zero because the supplier cannot force the downstream parties to buy the product.

The characteristic function for the coalition $v(1,2)$ is obtained considering the sum of $\Pi_1+\Pi_2$ because the coalition $\{1,2\}$ gets the payoff of both players together.

$$v(1,2)=\max_{p_1,p_2}\min_{w_1,w_2}\left[\Pi_1(p_1,p_2,w_1,w_2)+\Pi_2(p_1,p_2,w_1,w_2)\right]=$$
$$\max_{p_1,p_2}\min_{w_1,w_2}\left[p_1(\beta_1-\gamma_1 p_1)-w_1(\beta_1-\gamma_1 p_1)+p_2(\beta_2-\gamma_2 p_2)-w_2(\beta_2-\gamma_2 p_2)\right]$$

Looking for the minimum values of w_1,w_2 that minimize the function and replacing them, we obtain (9):

$$v(1,2)=\max_{p_1,p_2}\left[p_1(\beta_1-\gamma_1 p_1)-\frac{\beta_1}{\gamma_1}(\beta_1-\gamma_1 p_1)+p_2(\beta_2-\gamma_2 p_2)-\frac{\beta}{\gamma_2}(\beta_2-\gamma_2 p_2)\right]=\max_{p_1,p_2}\left[\Pi_1+\Pi_2\right]$$

We have to look for the values of p_1,p_2 that make the function maximum. Thus, we derivate with respect to p_1 *and* p_2. The first derivatives must be equal to 0 and the second ones must be negative to ensure the presence of a maximum.

$$\frac{\partial[\Pi_1+\Pi_2]}{\partial p_1}=2\beta_1-2\gamma_1 p_1=0 \qquad \frac{\partial^2[\Pi_1+\Pi_2]}{\partial p_1{}^2}=-2\gamma_1$$

$$\frac{\partial[\Pi_1+\Pi_2]}{\partial p_2}=2\beta_2-2\gamma_2 p_2=0 \qquad \frac{\partial^2[\Pi_1+\Pi_2]}{\partial p_2{}^2}=-2\gamma_2$$

These values are:

$$p_1=\frac{\beta_1}{\gamma_1} \qquad q_1=0$$

$$p_2=\frac{\beta_2}{\gamma_2} \qquad q_2=0$$

Replacing in equation (9) we get the characteristic function

$$v(\{1,2\})=v(1,2)=0 \qquad (10)$$

This is the payoff that will get the coalition between the downstream parties in the worst condition that the supplier is against them. The supplier will charge them the highest price and their response will be to put the highest prices and they will sell 0. The benefits they get are equal to zero in the worst condition. This equation points out the fact that if the downstream firms cooperate with each other, without any consideration of the third player (the supplier), they will be able to get 0. They cannot force the supplier to invest and sell them the product.

The characteristic function for the coalition $v(1,3)$ is:

$$v(1,3)=\max_{p_1,w_1,w_2}\min_{p_2}\left[\Pi_1(p_1,p_2,w_1,w_2)+\Pi_3(p_1,p_2,w_1,w_2)\right]=$$

$$\max_{p_1,w_1,w_2}\min_{p_2}\left[p_1(\beta_1-\gamma_1 p_1)+w_2(\beta_2-\gamma_2 p_2)-\lambda c_{s1}(\beta_1-\gamma_1 p_1)-\lambda c_{s2}(\beta_2-\gamma_2 p_2)\right]$$

Looking for the minimum value of p_2 that minimizes the function, we get:

$$v(1,3)=\max_{p_1,w_1,w_2}\left[p_1(\beta_1-\gamma_1 p_1)+w_2\left(\beta_2-\gamma_2\frac{\beta_2}{\gamma_2}\right)-\lambda c_{s1}(\beta_1-\gamma_1 p_1)-\lambda c_{s2}\left(\beta_2-\gamma_2\frac{\beta_2}{\gamma_2}\right)\right]$$

$$v(1,3)=\max_{p_1,w_1,w_2}\left[p_1(\beta_1-\gamma_1 p_1)-\lambda c_{s1}(\beta_1-\gamma_1 p_1)\right]=\max_{p_1,w_1,w_2}\left[\Pi_1+\Pi_3\right] \quad (11)$$

Now, we have to find the value of p_1 that maximizes (11)

$$\frac{\partial[\Pi_1+\Pi_3]}{\partial p_1}=\beta_1-2\gamma_1 p_1+\lambda c_{s1}\gamma_1=0 \qquad \frac{\partial^2[\Pi_1+\Pi_3]}{\partial p_1^2}=-2\gamma_1$$

$$p_1=\frac{\beta_1+\lambda c_{s1}\gamma_1}{2\gamma_1} \qquad q_1=\frac{\beta_1-\lambda c_{s1}\gamma_1}{2}$$

Replacing these values in the equation (11), we get the characteristic function

$$v(\{1,3\})=v(1,3)=\frac{(\beta_1-\lambda c_{s1}\gamma_1)^2}{4\gamma_1} \quad (12)$$

This is the payoff that the coalition between the downstream firm 1 and the supplier can get if the downstream firm 2 is assumed to oppose them. The supplier will produce $q_1=\frac{\beta_1-\lambda c_{s1}\gamma_1}{2}$ and the downstream will sell it at the price $p_1=\frac{\beta_1+\lambda c_{s1}\gamma_1}{2\gamma_1}$. The characteristic function implies that, under any circumstances, the downstream firm 1 and the supplier, together, are sure to obtain the least amount given by this equation. This result is the same as if the supplier and the downstream firm 1 were vertically integrated. It is necessary to note that this coalition makes sense if the investments are specific because, if the investments were general, the players neither need to form coalitions nor to be vertically integrated.

Doing the same for the coalition between the downstream firm 2 and the supplier, we get:

$$v(2,3)=\max_{p_1,w_1,w_2}\min_{p_1}\left[\Pi_2(p_1,p_2,w_1,w_2)+\Pi_3(p_1,p_2,w_1,w_2)\right]=$$
$$\max_{p_2,w_1,w_2}\min_{p_1}\left[p_2(\beta_2-\gamma_2 p_2)+w_1(\beta_1-\gamma_1 p_1)-\lambda c_{s1}(\beta_1-\gamma_1 p_1)-\lambda c_{s2}(\beta_2-\gamma_2 p_2)\right]$$

Looking for the minimum value of p_1 that minimizes the function, we get:

$$v(2,3)=\max_{p_2,w_1,w_2}\left[p_2(\beta_2-\gamma_2 p_2)+w_1\left(\beta_1-\gamma_1\frac{\beta_1}{\gamma_1}\right)-\lambda c_{s1}\left(\beta_1-\gamma_1\frac{\beta_1}{\gamma_1}\right)-\lambda c_{s2}(\beta_2-\gamma_2 p_2)\right]$$

$$v(2,3)=\max_{p_2,w_1,w_2}\left[p_2(\beta_2-\gamma_2 p_2)-\lambda c_{s2}(\beta_2-\gamma_2 p_2)\right]=\max_{p_2,w_1,w_2}\left[\Pi_2+\Pi_3\right] \quad (13)$$

Now, we have to find the value of p_2 that maximizes (13)

$$\frac{\partial\left[\Pi_2+\Pi_3\right]}{\partial p_2}=\beta_2-2\gamma_2 p_2+\lambda c_{s2}\gamma_2=0 \qquad \frac{\partial^2\left[\Pi_2+\Pi_3\right]}{\partial {p_2}^2}=-2\gamma_2$$

$$p_2=\frac{\beta_2+\lambda c_{s2}\gamma_2}{2\gamma_2} \qquad q_2=\frac{\beta_2-\lambda c_{s2}\gamma_2}{2}$$

Replacing these values in the equation (13), we get the characteristic function

$$v(\{2,3\})=v(2,3)=\frac{(\beta_2-\lambda c_{s2}\gamma_2)^2}{4\gamma_2} \quad (14)$$

This is the payoff that the coalition between the downstream firm 2 and the supplier can get if the downstream firm 1 acts against them. The supplier will produce $q_2=\frac{\beta_2-\lambda c_{s2}\gamma_2}{2}$ and the downstream will sell it at the price

$p_2 = \frac{\beta_2 + \lambda c_{s2}\gamma_2}{2\gamma_2}$. The characteristic function implies that, under any circumstances, the downstream firm 2 and the supplier, together, are sure to obtain the least amount given by this equation. As in the previous case, this result is the same as if the supplier and the downstream firm 2 were vertically integrated.

The characteristic function for the coalition $(1,2,3)$ is given by:

$$v(1,2,3) = \max_{p_1, p_2, w_1, w_2} \left[\Pi_1(p_1, p_2, w_1, w_2) + \Pi_2(p_1, p_2, w_1, w_2) + \Pi_3(p_1, p_2, w_1, w_2) \right] =$$

$$\max_{p_1, p_2, w_1, w_2} \left[p_1(\beta_1 - \gamma_1 p_1) + p_2(\beta_2 - \gamma_2 p_2) - \lambda c_{s1}(\beta_1 - \gamma_1 p_1) - \lambda c_{s2}(\beta_2 - \gamma_2 p_2) \right]$$

And now we must find the values p_1, p_2 that maximize the function.

$$\frac{\partial[\Pi_1 + \Pi_2 + \Pi_3]}{\partial p_1} = \beta_1 - 2\gamma_1 p_1 + \lambda c_{s1}\gamma_1 = 0 \qquad \frac{\partial^2[\Pi_1 + \Pi_2 + \Pi_3]}{\partial p_1^2} = -2\gamma_1$$

$$p_1 = \frac{\beta_1 + \lambda c_{s1}\gamma_1}{2\gamma_1} \qquad q_1 = \frac{\beta_1 - \lambda c_{s1}\gamma_1}{2}$$

$$\frac{\partial}{\partial p_2} = \beta_2 - 2\gamma_2 p_2 + \lambda c_{s2}\gamma_2 = 0 \qquad \frac{\partial^2}{\partial p_2^2} = -2\gamma_2$$

$$p_2 = \frac{\beta_2 + \lambda c_{s2}\gamma_2}{2\gamma_2} \qquad q_2 = \frac{\beta_2 - \lambda c_{s2}\gamma_2}{2}$$

Replacing these values in the equation we get that:

$$v(\{1,2,3\}) = v(1,2,3) = \frac{(\beta_1 - \lambda c_{s1}\gamma_1)^2}{4\gamma_1} + \frac{(\beta_2 - \lambda c_{s2}\gamma_2)^2}{4\gamma_2} \qquad (15)$$

This expression is the characteristic function of the grand coalition. This is the maximum payoff that the grand coalition, or total coalition, can achieve if they decide to cooperate with each other. If we compare this payoff with the benchmark, we can observe that we reach the first best if the players decide to cooperate all together. If we also compare this result with the non-cooperative solution, we can argue that the solution under cooperative framework is better than the one under non-cooperative framework where the benefits for the players are lower due to of the presence of double marginalization.

Once we have computed the characteristic function for all the coalitions, we also know that, due to the definition of the characteristic function given by Thomas (1984), if S and T are disjoint coalitions, we get:

$v(S \cup T) \geq v(S) + v(T),\ if\ S \cap T = \{\varphi\}$. Where φ stands for the empty set.

So, in our model, the following expressions are satisfied

$$v(1,2,3) \geq v(1) + v(2) + v(3)$$
$$v(1,2) \geq v(1) + v(2)$$
$$v(1,3) \geq v(1) + v(3)$$
$$v(2,3) \geq v(2) + v(3)$$
$$v(1,2,3) \geq v(1,2) + v(3)$$
$$v(1,2,3) \geq v(1,3) + v(2)$$
$$v(1,2,3) \geq v(2,3) + v(1)$$
$$v(1,2,3) + v(3) \geq v(1,3) + v(2,3)$$
$$v(1,2,3) + v(1) \geq v(1,2) + v(1,3)$$
$$v(1,2,3) + v(2) \geq v(2,3) + v(1,2)$$

This means that superadditivity holds for the characteristic function. We also know that the presence of superadditivity indicates that the introduction of one player into the coalition adds value to it. Therefore, this means that the marginal contribution of each player added to a coalition, is not null.

To solve this model, we use the core as the solution and then the Shapley Value that is the baricenter of the core. For a convex characteristic function, we determine how the benefits of the model can be split between the players under the negotiation process. In other words, we are going to solve the model taking into account the possible set of rewards that the players can reach. We will consider the entire problem to find the solution, given that the game is essential. That the game is essential implies that the characteristic function is superadditive and therefore, for two disjoint coalitions the value of the characteristic function of the union is strictly greater that the sum of the value of the individual characteristic functions. This is equivalent to say that $v(1) + v(2) + v(3) < v(1,2,3)$ (Myerson, 1991).

2.5- THE CORE AS A SOLUTION OF THE GAME

Having the characteristic function of the game of the two downstream firms and the supplier, it is important to find a suitable concept of solution. For this reason, first we choose the core as a possible solution. Previous to define the core, it is necessary to define an imputation[4]. An imputation in an *n*-person game with characteristic function v is a vector $x=(x_1, x_2, \ldots, x_n)$ satisfying the following:

(i) $\sum_{i=1}^{n} x_i = v(N);$

(ii) $x_i \geq v(i), \quad for\ i=1,2,\ldots,n$

Where x_i is obviously the ith player´s reward. The condition (i) is a Pareto optimality condition or the rationality of the grand coalition. $v(N)$ is the most the players can get out of the game when they all work together. So, for any possible set of individual rewards x_i ; we must have $\sum_{i=1}^{n} x_i \leq v(N)$. If this was a strict inequality, then, by working together, they could always share out the rewards so that everyone got more. The condition (ii) says that everyone must get as much as they could get if they played by themselves.

Once we had defined the imputation concept, we introduce the concept of the core (Gilles, 1953). The core of a game v , denoted by $C(v)$, is the set of imputations which are not dominated for any other coalition. This means that you can not find another imputation y and a coalition S such that

$$\sum_{i \in S} y_i \leq v(S)\ and\ y_i > x_i\ for\ all\ i \in S$$

Thus, if x is in the core, any coalition which forms, either says x is the best imputation for it. Notice there can be more than one imputation in the core. We provide a result which can be found in Thomas (1984).

Theorem: x is in the core if and only if

(i) $\sum_{i=1}^{n} x_i = v(N),$ and

(ii) $\sum_{i \in S} x_i \geq v(S)$ for all $S \subset N$

[4] Thomas, L.C., 1984. Games, Theory and Applications. Ellis Horwood Limited

Once the concepts of an imputation and the core are given, we can apply the core solution in our model. To apply the core, it is feasible to consider in a better way its 0-1 reduction. It must satisfy:

$$\bar{w}(S)=kv(S)+\sum_{i\in S} w_i \qquad (16)$$

Such that:

$\bar{w}(i)=0$;
$\bar{w}(1,2,3)=1$

Taking the characteristic functions of the model

$v(\{1\})=v(1)=0 \qquad v_2(\{2\})=v(2)=0 \qquad v(\{3\})=0$

$v(\{1,2\})=v(1,2)=0$

$$v(\{1,3\})=v(1,3)=\frac{(\beta_1-\lambda c_{s1}\gamma_1)^2}{4\gamma_1}$$

$$v(\{2,3\})=v(2,3)=\frac{(\beta_2-\lambda c_{s2}\gamma_2)^2}{4\gamma_2}$$

$$v(\{1,2,3\})=v(1,2,3)=\frac{(\beta_1-\lambda c_{s1}\gamma_1)^2}{4\gamma_1}+\frac{(\beta_2-\lambda c_{s2}\gamma_2)^2}{4\gamma_2}$$

Next we look for the value $\bar{w}(S)$. To simplify the mathematical calculation, we denote $\alpha=\frac{(\beta_1-\lambda c_{s1}\gamma_1)^2}{4\gamma_1}$ and $\tau=\frac{(\beta_2-\lambda c_{s2}\gamma_2)^2}{4\gamma_2}$.

Applying equation (16), we get:

$$\bar{w}(1)=kv(1)+w_1=k0+w_1=0$$

$$\bar{w}(2)=k0+w_2=0$$

$$\vec{w}(3)=kv(3)+w_3=k0+w_3=0$$

$$\bar{w}(1,2,3)=kv(1,2,3)+w_1+w_2+w_3=1$$

$$k=\frac{1}{v(1,2,3)}$$

$$\bar{w}(1,2)=kv(1,2)+w_1+w_2=0$$

$$\bar{w}(1,3)=kv(1,3)+w_1+w_3=\frac{v(1,3)}{v(1,2,3)}=\frac{\alpha}{\alpha+\tau}$$

$$\bar{w}(2,3)=kv(2,3)+w_2+w_3=\frac{v(2,3)}{v(1,2,3)}=\frac{\tau}{\alpha+\tau}$$

These equations are the characteristic functions reduced to 0-1. And once we have reduced the characteristic function, we can then apply the reduced core. It must satisfy the following:

$$x_i \geq 0$$
$$x_1 + x_2 \geq \bar{w}(1,2) = 0$$
$$x_1 + x_3 \geq \bar{w}(1,3) = \eta$$
$$x_2 + x_3 \geq \bar{w}(2,3) = \sigma$$
$$x_1 + x_2 + x_3 = 1 = \varsigma$$

Where η, σ and ζ denote the characteristic functions in the 0-1 reduced core. So, we can get the feasible set of payoffs of the model as the figure 2 shows.

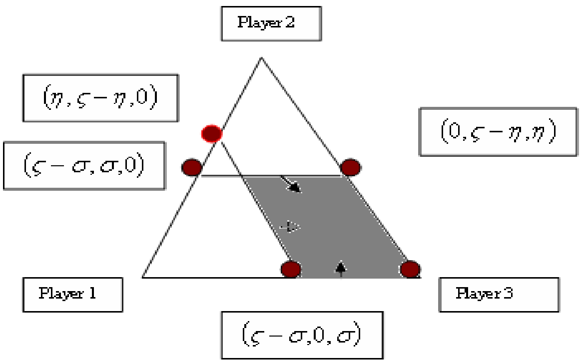

Figure 2: The reduced core solution

The painted area is the reduced core solution. Therefore all the points of the reduced core are possible solutions for the game played by the three agents under consideration. Moreover, under the condition that $\eta + \sigma \leq \eta * \sigma$ in the reduced game, the game is convex in the sense $v(S) + v(T) \leq v(S \cup T) + v(S \cap T)$ for all S and T[5] subsets of $\{1,2,3\}$. The last property is also valid for non-reduced games.

In the case that the characteristic function is convex, then both the reduced and non-reduced core are non-empty, and have a regular structure. In particular, there is the Shapley value which is the center of gravity of the extreme points of

5 Shapley, Lloyd, 1971. Cores of convex games. International Journal of Game Theory 1, 11-26

the core. If the core is only a point, the Shapley value is unique and coincides with the core (Shapley, 1965).

2.6-THE SHAPLEY VALUE

A solution concept for the game under consideration is the Shapley value (Peleg and Sudhölter, 2003) and (Branzei and Tijs, 2005). Shapley (1953) looked at what each player could reasonably get before the game has begun. He put three axioms, which he called $\varphi_i(v)$, player i´s expectation in a game with a characteristic function v , should satisfy the following:

S1: $\varphi_i(v)$ is independent of the labeling of the players. If π is a permutation of 1, 2,...., n and $\pi\ v$ is the characteristic function of the game, with the players numbers permuted by π , then $\varphi_{\pi(i)}(\pi v)=\varphi_i(v)$.

S2: The sum of the expectations should equal the maximum available from the game, so

$$\sum_{i=1}^{n} \varphi_i(v)=v(N)$$

S3: If u, v are the characteristic functions of two games, $u+v$ is the characteristic function of the game playing both games together. φ must satisfy $\varphi_i(u+v)=\varphi_i(u)+\varphi_i(v)$.

Given these assumptions, Shapley proved the following theorem:

Theorem. There is only one function which satisfies S1, S2 and S3, namely:

$$\varphi_i(v)=\sum_{S:i\in S} \frac{(¿S-1)!(n-¿S)!}{n!}\left(v(S)-v(S-\{i\})\right)$$

Where the summation is over all coalitions S which contain player i and # S is the number of players in the coalition S . $\varphi_i(v)$ is called the Shapley value. The quantity φ_i may be interpreted as the “equity value” associated with the position of the i-th player in the game (Shapley, 1965).

In our model, the Shapley value is:

For downstream firm 1,

$$\varphi_1(v)=\frac{1}{2}\frac{(\beta_1-\lambda c_{s1}\gamma_1)^2}{4\gamma_1}$$

For downstream firm 2,

$$\varphi_2(v)=\frac{1}{2}\frac{(\beta_2-\lambda c_{s2}\gamma_2)^2}{4\gamma_2}$$

For the supply firm

$$\varphi_3(v)=\frac{1}{2}\left(\frac{(\beta_1-\lambda c_{s1}\gamma_1)^2}{4\gamma_1}+\frac{(\beta_2-\lambda c_{s2}\gamma_2)^2}{4\gamma_2}\right)$$

And the total value created is equal to $\frac{(\beta_1-\lambda c_{s1}\gamma_1)^2}{4\gamma_1}+\frac{(\beta_2-\lambda c_{s2}\gamma_2)^2}{4\gamma_2}$. As it can be checked, this total value is equal to the first best and to the value obtained when the three firms cooperate with each other. That is, the grand coalition's characteristic function.

The Shapley value indicates how the total benefits can be split in a fair way. We can see that the supplier gets more benefits compared with the benefits that each downstream firm gets. This indicates what we posed in the previous section that the cooperative game theory attempts to answer how the total value is divided up among various players. We remark that this answer will depend on the bargaining power of the players, and this is determined by which players are most needed. In this model, the supplier is the most needed because the firms have incentives to merger or to be vertically integrated with the supply firm, for the reason that a merger or the vertical integration ensures them that the production will have the required quality. In other words, the bargaining power of the supplier relies on the specificity of the investments because it is this specificity what makes the cooperation credible. If the investments were general, non-specific, the cooperation framework would not be enforceable because each player could sell or produce outside the relationship. Such bargaining power is measured by the Shapley value and it is what makes possible the cooperation among the three players, achieving a Pareto optimum solution. Thus, this higher supplier's bargaining power is what makes the upstream-downstream relationship an enforceable one. As a result of this, if the the three firms commit to cooperate and they distribute the benefits as the

Shapley value indicates, at time 0, they will decide the quantity level and the prices that maximize total welfare.

3- CONCLUSIONS

In this paper we have developed a model within the upstream-downstream relationship that attempts to find an alternative solution to vertical integration. Our contribution is twofold: first, a mathematical part where we show how to apply the maximin to compute the characteristic functions of the model; and second, an economic contribution, where we explain how cooperation can lead to organizational forms that generate more value. In fact, we set another way to treat the upstream-downstream relationship under incomplete contracts and a cooperative framework, providing an alternative to vertical integration.

As a solution, vertical integration presents some problems and it seems to be a poor solution under certain circumstances. In particular, the existence of these failures made us wonder if there could be another way to solve those problems that vertical integration did not solve. More specifically, what happens if the supply firm does not want to be vertically integrated? What happens if the firms decide to cooperate instead of being vertically integrated under the presence of specific investments? To answer these questions we have developed a model where the firms cooperate with each other. We find that, under cooperation, the total value generated is Pareto optimum if they cooperate and distribute the benefits through the Shapley value solution. The Shapley value measures the power that firms enjoy in the bargaining process, where bargaining power under cooperative game theory is defined in terms of how much each player is needed. The result of our model is that the supplier's bargaining power is determined by the specificity of the investments and this makes the cooperation among the firms possible, and the upstream-downstream relationship enforceable. As a consequence, the first best, that maximizes the total welfare, can be achieved.

In this paper, we have presented a one-shot supply transaction. We want to consider further developments of this model, with n-periods and more participants, in our future research. In such scenarios, one must include relational contracts and the reputation of the firms, increasing the complexity of the analysis. We are also interested in the use of biform games, which combine both the non-cooperative game theory and the cooperative game approach in the same analysis. We believe that more efforts should be implemented in this direction to achieve a better understanding of organizational forms.

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